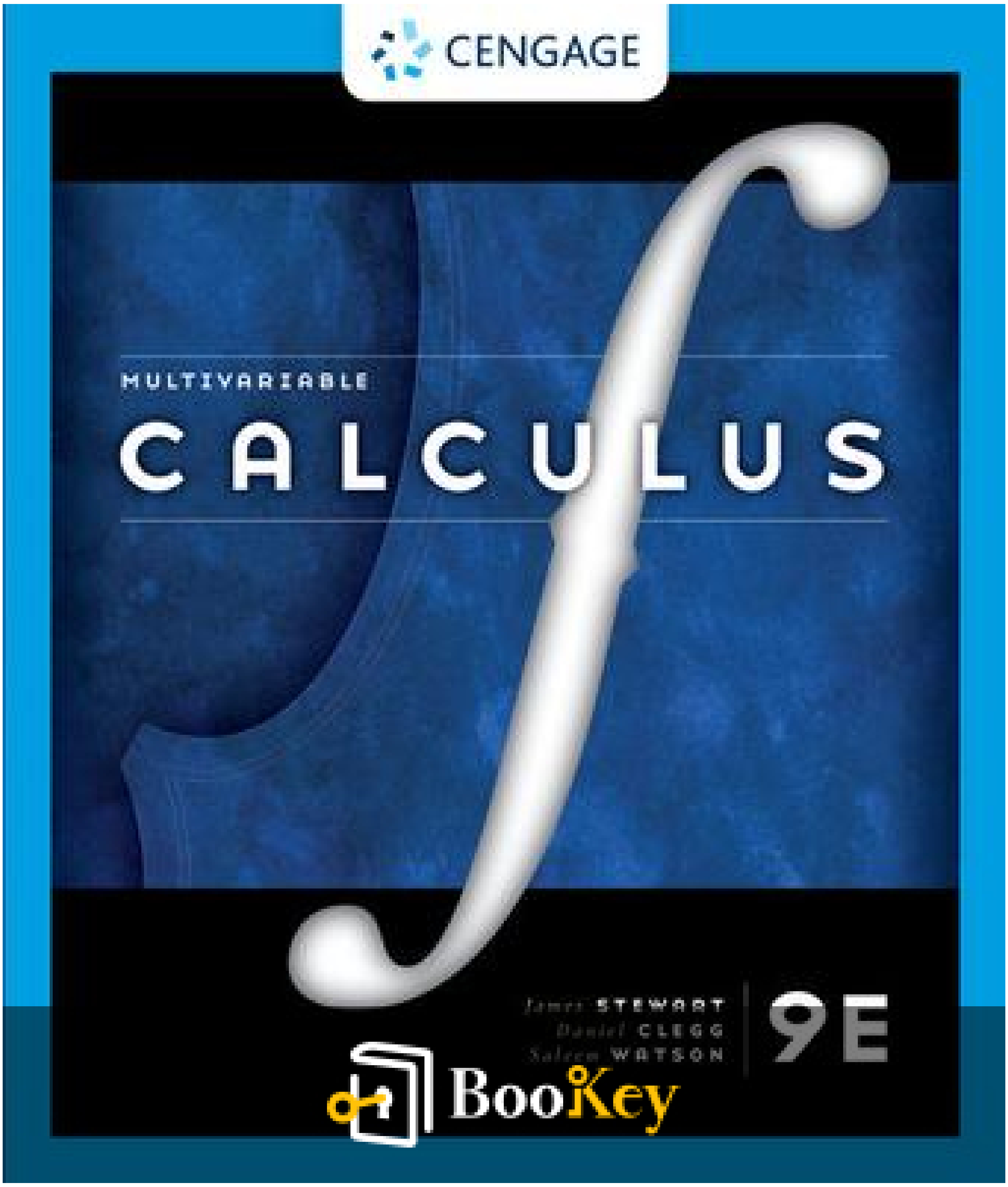
Multivariable Calculus PDF (Limited Copy)

James Stewart



Multivariable Calculus Summary

"Exploring the Math of Change in Multiple Dimensions."

Written by Books1

About the book

Dive into the captivating world of multivariable calculus with James Stewart's authoritative book, where complex concepts are unraveled with clarity and precision. Developed for learners eager to transcend the boundaries of single-variable calculus, this book serves as an essential guide through the intricacies of three-dimensional calculus, vector analysis, and partial derivatives. Stewart's pedagogical approach is both rigorous and accessible, offering intuitive explanations and insightful examples that make the challenging territory of multivariable calculus navigable and enthralling. Whether you're a student seeking to advance your calculus prowess or an enthusiast with a thirst for mathematical depth, Stewart's carefully curated text promises not just to inform, but to engage, enrich, and inspire. Embark on this intellectual journey where calculus becomes more than a subject—it's an exploration of the mathematical elegance that underpins our multidimensional universe.

About the author

James Stewart was a prolific mathematician and esteemed author renowned for his contributions to the field of calculus, most notably through his widely-used textbook series. Born in Toronto, Canada, in 1941, Stewart demonstrated an early aptitude for mathematics which later blossomed into a formidable academic career. Upon completing his bachelor's degree from the University of Toronto, he received his Ph.D. from Stanford University under the supervision of George Polya, a prominent mathematician known for his work in problem-solving. Stewart's passion for teaching led him to positions at several prestigious educational institutions, culminating in a long-standing tenure at McMaster University. His authoritative texts on calculus, starting with "Calculus: Concepts and Contexts," have been instrumental in shaping the curriculum and understanding of calculus concepts for countless students worldwide, marking him as a transformative figure in mathematics education. He devoted his life to making complex mathematical ideas accessible to learners, ensuring his work leaves an enduring legacy in the academic landscape.

Summary Content List

Chapter 1 Summary: Parametric Equations and Polar Coordinates

Chapter Summary for Chapter 10: Parametric Equations and Polar Coordinates

Introduction to Parametric Equations and Polar Coordinates:

Parametric equations and polar coordinates offer alternative methods for describing curves, expanding beyond traditional Cartesian coordinates. Instead of expressing a curve solely as y as a function of x, parametric equations use a parameter t to describe x and y as functions of t. Conversely, polar coordinates provide a method to describe curves using a radius and angle, convenient for curves with circular or radial symmetry. This section lays the foundation for deeper exploration of curves, starting with more complex curve types such as cycloids and cardioids.

Section 10.1: Curves Defined by Parametric Equations

Parametric equations involve defining both x and y in terms of a third variable, the parameter t, allowing for the description of curves that might not be easily represented in standard Cartesian form. This section covers the basic setup and algebra needed to work with parametric equations, including transforming between parametric and Cartesian forms, sketching parametric curves, and understanding the movement of points along these curves.

Examples and Applications:

Examples illustrate various parametric equations, including cycloids traced by points on a rolling circle and other special curves. Calculations showing elimination of the parameter t to convert parametric equations into Cartesian equations are given. The section introduces the concept of direction and speed along a parametric curve, which are especially useful for describing motion along paths.

Section 10.2: Calculus with Parametric Curves

This section extends calculus principles to parametric curves, covering differentiation, finding tangents and normals, arc lengths, and areas under these curves. It illustrates how to determine tangents to parametric curves and derive relevant functions for slope, demonstrating concavity and curve behavior. Speed and arc length formulas in terms of parametric equations are provided, deepening the comprehension of motion along curves.

Section 10.3: Polar Coordinates

Polar coordinates represent points based on their distance from a fixed point (the pole) and an angle from a fixed direction (the polar axis). This section explores the relationships between Cartesian and polar coordinates and presents graphical representations of equations expressed in polar form. Particular polar curves, such as circles and cardioids, are discussed, providing visual examples of how they can offer simpler equations for certain shapes.

Symmetry and Graphing in Polar Coordinates:

Graphing polar curves involves recognizing symmetries, such as symmetry about the polar axis or pole. The section includes discussions on sketching and converting polar equations to Cartesian form, aiding in understanding how these curves function and are represented in multiple mathematical contexts.

Section 10.4: Calculus in Polar Coordinates

The principles of calculus are applied to polar curves. Calculating areas within polar curves involves setting up integrals in polar form, while arc length calculations are adapted to polar coordinates through specialized formulas. Techniques for determining tangent lines to these curves expand the methodological applications, allowing deeper analysis of curves with respect to their geometric properties.

Section 10.5: Conic Sections

Conics, including parabolas, ellipses, and hyperbolas, are introduced as curves derived from specific geometric definitions. Each type is explored with specific equations and properties, highlighting their applications and features, such as their reflection properties and practical uses in optics and astronomy. The section delves into deriving equations from foci and directrices and solving for vertices, foci, and axes.

Chapter Conclusion:

Overall, Chapter 10 bridges fundamental geometric and algebraic concepts with advanced calculus applications, emphasizing parametric and polar forms as valuable tools for expressing and analyzing curves beyond what's possible with Cartesian coordinates alone. It provides rigorous approaches to understanding and applying these systems to practical mathematical and real-world contexts.

Chapter 2 Summary: Sequences, Series, and Power Series

Summary of the Chapters:

Chapter 11: Sequences, Series, and Power Series

Sequences

- A sequence is defined as a function whose domain is a set of natural numbers. They are represented as ordered lists of numbers and are fundamental in calculus.
- Examples include Newton’s method for approximating function zeros and Zeno's paradox illustrating infinite sequencing.
- Infinite sequences have terms a_n represented as a function. The concept of a limit of a sequence is introduced, where the sequence approaches a single number, L, as n becomes infinite.
- Convergent sequences tend toward a limit, while divergent ones do not.

Series

- A series is the sum of a sequence of numbers. It can be finite or infinite.
- Convergence of a series depends on whether its sequence of partial sums

has a limit.

- Geometric series and harmonic series are special types, with specific convergence criteria.
- The Test for Divergence states if the terms of a series do not tend to zero, the series is divergent.
- Important theorems regarding series include the Integral Test and Comparison Tests, which provide methods for determining convergence or divergence.

Power Series

- A power series is an infinite series where each term contains a power of a variable x.
- The convergence of a power series is determined by its interval and radius of convergence.
- Power series can be manipulated through differentiation and integration under certain conditions, maintaining their radius of convergence.

Taylor and Maclaurin Series

- Taylor series represent a function as an infinite sum of terms calculated from the values of its derivatives at a single point.
- Maclaurin series is a special Taylor series centered at zero.
- They provide practical ways to approximate complex functions through

polynomials, useful in computations and theoretical physics.
- Specific series expansions for functions such as e^x, sin x, cos x, and ln(1+x) display essential characteristics and ranges of convergence.

Applications

- Taylor polynomials serve in approximating functions and are integral in physics to simplify complex equations and in engineering for modeling purposes like optics and mechanical systems.
- The importance of assessing the accuracy of polynomial approximations and understanding the implications on various scientific models is explored.

Overall, these chapters dive deep into understanding the nature of sequences and series, exploring their convergence properties, and using them to approximate functions, which is fundamental both in theoretical mathematics and practical applications, spanning physics, engineering, and computing.

Topic	Summary
Sequences	A function with domains as natural numbers representing ordered lists. Important for understanding convergence toward a limit, L, with examples like Newton's method and Zeno's paradox.
Series	Sum of sequential numbers, either finite or infinite. Convergence is if its partial sums have a limit. Includes geometric and harmonic series, with tests like the Integral and Comparison Tests for checking convergence.

Topic	Summary
Power Series	Infinite series with powers of a variable x, with convergence determined by interval and radius. They can often be manipulated by differentiation and integration.
Taylor and Maclaurin Series	Methods to express functions as infinite sums of derivatives. Useful for approximating complex functions; Maclaurin series is a Taylor series centered at zero.
Applications	Using Taylor polynomials to approximate functions in various contexts like physics and engineering, emphasizing the importance of accuracy.

Chapter 3 Summary: Vectors and the Geometry of Space

These chapters from a mathematics textbook explore the concepts of vectors and geometry in three-dimensional space, focusing on the mechanics and applications of vectors, equations of lines and planes, and forms of three-dimensional surfaces.

Chapter Summary:

- Vectors and Geometry of Space:

- Vectors are quantities with both magnitude and direction. They are used to represent forces, velocity, and other directional quantities in physics.
- Vectors can be added or multiplied by scalars to scale their magnitude. The sum and difference of vectors, as well as their scalar multiples, allow for various mathematical operations that represent physical scenarios.
- The dot product and cross product are essential operations. The dot product gives a scalar and is related to the cosine of the angle between vectors, typically used to find projection components and work done by forces. The cross product results in a vector perpendicular to the original two vectors, with applications in calculating torque and areas of parallelograms.

- Equations of Lines and Planes:

- Lines in space can be described using vector and parametric equations based on a point and a directional vector. These equations characterize the geometrical path of a line in a 3D space.
- A plane in three spaces is defined through a point on the plane and a normal vector perpendicular to the plane. Equations of planes take the form $ax + by + cz + d = 0$, where d is derived from the specific point contained on the plane.
- The intersection of lines and planes involves solving systems of equations that represent each line or plane. Skew lines demonstrate that lines in space that neither intersect nor are parallel can exist, lying on different planes.

- Cylinders and Quadric Surfaces:

- Cylinders in space are defined as surfaces formed by aligning parallel lines to a given curve. In 3D, they can extend infinitely along an axis and include forms such as parabolic and elliptical cylinders.
- Quadric surfaces are 3D counterparts of conic sections in 2D, encompassing a range of forms like ellipsoids, hyperboloids, and paraboloids. Each is described by specific standard equations that derive their unique shapes and properties.
- The chapter explores how translation and transformation can simplify complex quadric surface equations into standard forms for easier interpretation and sketching.

This summary clarifies the relationship between vectors, lines, planes, and three-dimensional surfaces. Mathematical techniques involved in these chapters are foundational for fields like physics, engineering, and various applications in 3D modeling and computer graphics.

Critical Thinking

Key Point: Vectors and their application

Critical Interpretation: As you delve into understanding vectors, you are essentially learning the language of spatial interaction. From achieving balance in your personal life to navigating challenges in your career, vectors remind you that every direction counts and that multiple forces often shape your path. Just as vectors describe force, velocity, and direction in physics, you are inspired to see your life's journey as a series of both distinct and interconnected vectors. Your strengths become magnitudes to leverage, and various pursuits intertwine as directional pathways leading you forward. Even when forces seem opposing (when you multiply vectors using the dot and cross products), there lies underlying potential for remarkable outcomes, teaching you about achieving harmony amidst perceived chaos, guiding your way through life's complex, three-dimensional space.

Chapter 4: Vector Functions

Chapter 13 delves into the study of vector functions and their applications in describing motion through space, building on the foundation of real-valued functions used in previous sections. This chapter offers essential insights into vector calculus, making it crucial for understanding the laws of planetary motion, particularly Kepler's laws.

The chapter begins with an introduction to vector functions and their role in defining paths in space, detailing that a vector function assigns a vector to each real number in its domain. A vector function typically has three component functions, which are real-valued, making it possible to represent the motion of objects in three-dimensional space.

Following this, the concept of limits and continuity of vector functions is introduced similarly to real-valued functions. A vector function is continuous if the limits of its component functions exist and match the function's value at that point. A critical aspect of vector functions is their ability to represent space curves—defined by parametric equations that depict the movement of a particle in space.

The chapter proceeds to discuss derivatives and integrals of vector functions and their significance in determining the behavior of curves and objects in motion. The derivative of a vector function is interpreted geometrically as the tangent vector of the space curve, guiding us to understand velocity and acceleration in the context of motion.

Key differentiation rules for vector functions reflect those for real-valued functions, aiding the computation of derivatives for complex vector functions. The integration of vector functions follows a similar component-wise approach, allowing the determination of motion through definite and indefinite integrals.

Moreover, the chapter introduces the arc length of space curves, which is determined by integrating the magnitude of the derivative of the vector function—essentially summing the speed over time to compute the distance travelled.

The idea of curvature is refined further, measuring how rapidly a curve changes direction, which is crucial in celestial mechanics for understanding the orbits of planets. Both components of acceleration—tangential and normal—are explored, providing insight into components responsible for speed change and directional shifts, respectively.

In the context of orbital mechanics, Kepler's laws are dissected. The First Law, concerning elliptical planetary orbits, is addressed through vector calculus, while the Second Law, involving areas swept out over time, highlights the constant areal velocity of planetary orbits, linked to angular momentum. The Third Law's insight into the proportional relationship between a planet's orbital period and the cube of the length of its orbit's semi-major axis is grounded in gravitational principles.

Overall, this chapter lays the mathematical groundwork essential for studying motion in physics, providing tools for analyzing trajectories, forces, and the intricate dance of celestial bodies in space.

Chapter 5 Summary: Partial Derivatives

Chapter 14 of the book discusses various aspects and applications of partial derivatives, which are essential in understanding the calculus of functions of multiple variables. Here's a summary of the key sections:

Partial Derivatives and Functions of Several Variables

- **Functions of Several Variables**: These functions can be analyzed verbally, numerically, algebraically, and visually. For example, temperature depends on longitude and latitude, or the volume of a cylinder depends on its radius and height.
- **Graphs and Level Curves**: Graphs of such functions represent surfaces, with each point on a surface corresponding to a value of the function. Level curves (or contour maps) help visualize the function's behavior, showing where the function has constant values.

Limits and Continuity

- **Limits**: The limit of a function of two variables as a point approaches a specific value examines how the function behaves near that point. The existence of the limit is sometimes path-dependent.
- **Continuity**: A function of two variables is continuous at a point if the limit at that point equals the function's value. Continuity across an entire domain means there's no break in the surface the function defines.

Calculating Partial Derivatives

- **Partial Derivatives**: They measure a function's rate of change concerning one variable while holding others constant. This concept extends to higher dimensions as well. For geometric or real-world interpretations, partial derivatives can represent slopes of tangent lines on surfaces.
- **Higher Derivatives**: Second-order and higher partial derivatives provide further information about the function's curvature and behavior.

Tangent Planes and Linear Approximations

- **Tangent Planes**: For a function of two variables, the tangent plane at a point approximates the function near that point. The equation of the tangent plane is derived using partial derivatives.
- **Linear Approximation**: This is a first-order Taylor approximation, using the tangent plane to approximate the function near the point of tangency.

Chain Rule and Implicit Differentiation

- **Chain Rule**: This extends to functions of several variables, allowing computation of derivatives of composite functions. Different cases handle different formations of dependent variables.
- **Implicit Differentiation**: This process involves using the Chain Rule to differentiate functions defined implicitly.

Directional Derivatives and the Gradient Vector

- **Directional Derivatives**: These measure the rate at which a function changes in any specified direction, generalizing partial derivatives.
- **Gradient Vector**: This vector points in the direction of the maximum rate of increase of the function and is perpendicular to level curves/surfaces.

Maximum and Minimum Values
- **Critical Points**: Points where partial derivatives are zero are examined to determine if they are maxima, minima, or saddle points, using the Second Derivatives Test.
- **Absolute Extrema**: The method involves evaluating the function at critical points and boundaries.

Lagrange Multipliers
- **Maxima/Minima with Constraints**: Lagrange's method helps find extrema of a function subject to constraints by introducing auxiliary variables, the multipliers.

Applications and Special Topics
- Various applied projects and problems extend these concepts to real-world scenarios, such as optimizing designs or systems.

Chapter 14 serves as a comprehensive guide to understanding and applying calculus to functions of multiple variables, showcasing both theoretical and practical aspects through diverse examples and problems.

Section Title	Description
Functions of Several Variables	Analyzed through verbal, numerical, algebraic, and visual methods. Examples include temperature as a function of longitude and latitude, or cylinder volume depending on its radius and height.
Graphs and Level Curves	Graphs represent surfaces where points correspond to function values, while level curves show where the function maintains constant values.
Limits and Continuity	Concerns the examination of function limits and continuity at points. Continuity signifies no breaks in the defined function surface.
Calculating Partial Derivatives	Measures the rate of change of a function relative to one variable while keeping others constant. Higher derivatives provide insights into curvature.
Tangent Planes and Linear Approximations	Tangent planes approximate functions near a point. Linear approximation utilizes the tangent plane for function approximation near a tangency point.
Chain Rule and Implicit Differentiation	Chain Rule allows derivative calculations in composite functions; implicit differentiation uses Chain Rule on functions defined implicitly.
Directional Derivatives and the Gradient Vector	Directional derivatives determine the rate of function change in a given direction, while the gradient vector points towards the maximum rate of increase.
Maximum and Minimum Values	Identifies critical points to determine maxima, minima, or saddle points using the Second Derivatives Test. Absolute extrema evaluate functions at critical points and boundaries.
Lagrange Multipliers	Used to find function extrema with constraints through auxiliary variables or multipliers.
Applications and Special Topics	Application projects and problems extend concepts, aiding in optimization tasks.

Chapter 6 Summary: Multiple Integrals

In Chapter 15 on Multiple Integrals, the text extends the idea of definite integration to functions of two or three variables via double and triple integrals. These integrals are then employed to calculate volumes, masses, and centroids of more complex regions compared to earlier chapters. Notably, double integrals are particularly useful for computing probabilities involving two random variables.

The chapter discusses using polar coordinates to simplify the computation of double integrals in specific regions. In three-dimensional contexts, cylindrical and spherical coordinates are introduced to streamline the evaluation of triple integrals over common solid regions.

1. **Double Integrals**: A double integral is used to find the volume under a surface over a region in the xy-plane, first by dividing the region into smaller rectangles and then summing the volumes of columns over these rectangles. The transformation to polar coordinates is explored for circular regions, making integration simpler due to the symmetry.

2. **Triple Integrals**: Extending double integrals, triple integrals allow for the computation of volumes in three-dimensional space, as well as other properties like mass and inertia for solids bounded in three-dimensional regions. Cylindrical and spherical coordinates are especially useful for

objects with rotational or spherical symmetry, providing simpler forms for integration over solid regions.

3. **Applications**: Multiple integrals aid in various physical applications—calculating mass when density is variable, finding centers of mass, computing moments of inertia, and dealing with probability distributions of multiple variables.

4. **Change of Variables**: This technique is crucial for simplifying the evaluation of double and triple integrals by transforming the region of integration into one where computation is more straightforward. The Jacobian determinant plays a key role here, facilitating the transformation from one set of variables to another, akin to substitution in single-variable calculus.

The comprehensive use of multiple integrals and their transformation into polar, cylindrical, and spherical coordinates underscores their versatility and power in solving complex problems in mathematics, physics, and engineering. These integrals, beyond computing volumes and masses, extend to interpreting more abstract concepts like probability distributions and moments of inertia, demonstrating their foundational role in various fields.

Section	Description

Section	Description
Introduction to Multiple Integrals	Expanding definite integration to functions of two or three variables with double and triple integrals. Utilized for more complex regions.
Double Integrals	Used to compute the volume under a surface over an xy-plane region, using rectangles. Transformation to polar coordinates simplifies computation in circular regions.
Triple Integrals	Used to calculate volumes and other properties in three dimensions. Cylindrical and spherical coordinates simplify problems with rotational or spherical symmetry.
Applications	Cover physical applications such as variable density mass calculation, centers of mass, moments of inertia, and probability distributions of multiple variables.
Change of Variables	Technique to simplify evaluation of integrals by transforming the integration region. Relies on the Jacobian determinant, analogous to substitution in single-variable calculus.
Conclusion	Emphasizes the versatility of multiple integrals in solving complex problems across mathematics, physics, and engineering via various coordinate transformations.

Chapter 7 Summary: Vector Calculus

Chapter 16 Summary: Integrals of Vector Fields

This chapter delves into the calculus of vector fields and the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem, exploring how they extend the Fundamental Theorem of Calculus to higher dimensions. A vector field assigns a vector to each point in space, useful for modeling diverse phenomena such as gravity, electromagnetism, and fluid flow.

Vector Fields:

Examples include velocity fields like wind patterns, ocean currents, and airflow. A vector field is called conservative if it is the gradient of a scalar function. The curl and divergence of a vector field measure its tendency to rotate and spread out, respectively. A conservative vector field has a curl of zero.

Integrals of Scalar Functions Over Curves (Line Integrals):

The line integral of a scalar function summed along a curve yields work done by a force field or mass of a wire with variable density. Line integrals of vector fields (e.g., force fields) calculate work.

Fundamental Theorem for Line Integrals:

The line integral of a gradient vector field only depends on its values at the endpoints of the curve, encapsulating the net change along the curve. If a line integral is path-independent, the vector field is conservative.

Green's Theorem:

Relates a line integral around a simple closed curve to a double integral over the region it encloses, providing a way to compute areas and work in planar fields by converting between boundary and region integrals.

Curl and Divergence in Vector Calculus:

Explore the curl, a vector representing the rotational tendency in a field, and divergence, a scalar indicating how much a field spreads or converges. They facilitate the understanding of flow and rotation in fields.

Parametric Surfaces and Their Areas:

Surfaces parameterized with vectors map two variables to points in space, essential for computing surface integrals. The area involves integrating the magnitude of the cross product of partial derivatives over the parameter domain.

Surface Integrals:

Measure how a surface absorbs or emits quantities, like fluid flow through a surface. The orientation of a surface is crucial for defining its surface integral, offering insights into the flux of a vector field across the surface.

Stokes' Theorem:

Extends Green's Theorem to surfaces and their boundaries in three dimensions. It equates the surface integral of the curl over a surface to the line integral over its boundary, providing a powerful tool for calculating circulation and flux.

Divergence Theorem:

Connects the flux across a closed surface to the triple integral of the divergence over the volume it encloses, pivotal for physical laws involving conserved quantities like mass, energy, and charge.

Overall, the chapter equips students with essential tools for modeling and solving problems involving vector fields, essential for fields like electromagnetism, fluid dynamics, and engineering analysis, by translating between local behaviors and global properties.

Critical Thinking

Key Point: Fundamental Theorem for Line Integrals

Critical Interpretation: Imagine being on a winding path, unsure of the journey's ups and downs. Just like the Fundamental Theorem for Line Integrals reveals how the journey is solely determined by your starting point and destination in a conservative vector field, it speaks to life's paths as well. Often, it's not the twists, turns, or obstacles that define our journey, but where we begin and where we aim to finish. In life, keeping our eyes on the end goal, similar to understanding the net change between endpoints, can inspire resilience during challenges. This realization empowers you to navigate life's complexities with a focus on your desired outcomes rather than being deterred by interim difficulties.